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On dynamical gluon mass generation

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Abstract. The effective gluon propagator constructed with the pinch technique is governed by a Schwinger-Dyson equation with special structure and gauge properties, that can be deduced from the correspondence with the background field method. Most importantly the non-perturbative gluon self-energy is transverse order-by-order in the dressed loop expansion, and separately for gluonic and ghost contributions, a property which allows for a meanigfull truncation. A linearized version of the truncated Schwinger-Dyson equation is derived, using a vertex that satisfies the required Ward identity and contains massless poles. The resulting integral equation, subject to a properly regularized constraint, is solved numerically, and the main features of the solutions are briefly discussed.

PACS. 12.38.Lg Other nonperturbative calculations – 12.38.Aw General properties of QCD (dynamics, confinement, etc.)

It is well known that one of the main theoretical problems when dealing with Schwinger-Dyson (SD) equations is that they are built out of unphysical off-shell Green's functions; thus, the extraction of reliable physical information depends crucially on delicate all-order cancellations, which may be inadvertently distorted in the process of the truncation. The truncation scheme based on the pinch technique (PT) [1,2] implements a drastic modification at the level of the building blocks of the SD series. The PT enables the construction of new, effective Green's functions endowed with very special properties; most importantly, they are independent of the gauge-fixing parameter, and satisfy QED-like Ward identities (WI) instead of the usual Slavnov-Taylor identities. The upshot of this approach would then be to trade the conventional SD series for another, written in terms of the new Green's functions, and then truncate this new series, by keeping only a few terms in a "dressed-loop" expansion, maintaining exact gauge-invariance. Of central importance in this context is the connection between the PT and the Background Field Method (BFM), a special gauge-fixing procedure that preserves the symmetry of the action under ordinary gauge transformations with respect to the background (classical) gauge field $\widehat{A}^a_\mu$. As a result, the background n-point functions satisfy QED-like all-order WIs. The connection between PT and BFM, which is known to persist to all orders (last two articles in [2]), affirms that the (gauge-independent) PT effective n-point functions coincide with the (gauge-dependent) BFM n-point functions provided that the latter are computed in the Feynman gauge. In this paper we report recent progress on the issue of gluon mass generation in the PT-BFM scheme [3].

We first define some basic quantities. There are two gluon propagators appearing in this problem, $\widehat{\Delta}_{\mu\nu}(q)$ and $\Delta_{\mu\nu}(q)$, denoting the background and quantum gluon propagator, respectively. Defining $P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$, we have that $\widehat{\Delta}_{\mu\nu}(q)$, in the Feynman gauge is given by

$$\widehat{\Delta}_{\mu\nu}(q) = -i\left[P_{\mu\nu}(q)\widehat{\Delta}(q^2) + \frac{q_\mu q_\nu}{q^4}\right], \qquad (1)$$

The gluon self-energy, $\widehat{\Pi}_{\mu\nu}(q)$, has the form $\widehat{\Pi}_{\mu\nu}(q) = P_{\mu\nu}(q)\,\widehat{\Pi}(q^2)$, and $\widehat{\Delta}^{-1}(q^2) = q^2 + i\widehat{\Pi}(q^2)$. Exactly analogous definitions relate $\Delta_{\mu\nu}(q)$ with $\Pi_{\mu\nu}(q)$.

As is widely known, in the conventional formalism the inclusion of ghosts is instrumental for the transversality of $\Pi^{ab}_{\mu\nu}(q)$, already at the level of the one-loop calculation. On the other hand, in the PT-BFM formalism, due to new Feynman rules for the vertices, the one-loop gluon and ghost contribution are individually transverse [4].

As has been shown in [3], this crucial feature persists at the non-perturbative level, as a consequence of the simple WIs satisfied by the full vertices appearing in the diagrams of fig. 1, defining the BFM SD equation for $\widehat{\Delta}_{\mu\nu}(q)$ [5]. Specifically, the gluonic and ghost sector are separately transverse, within each individual order in the dressed-loop expansion.

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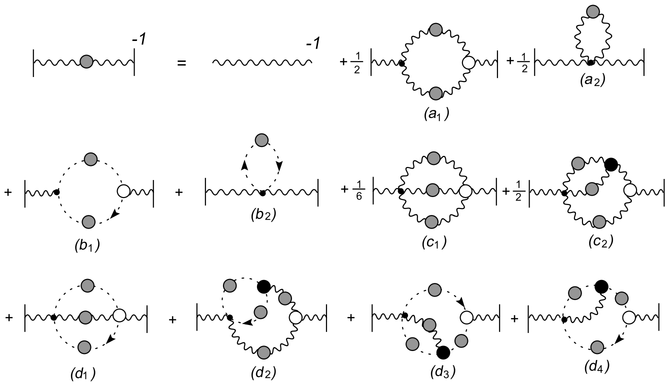

Fig. 1. The SD equation for the gluon propagator in the BFM. All external legs (ending with a vertical line) are background gluons, wavy lines with grey blobs denote full-quantum gluon propagators, dashed lines with grey blobs are full-ghost propagators, black dots are the BFM tree-level vertices, black blobs are the full conventional vertices, and white blobs denote full three- or four-gluon vertices with one external background leg.

Let us demonstrate this property for graphs (a_1) and (a_2), given by

$$\widehat{\Pi}^{ab}_{\mu\nu}(q)\big|_{a_1} = \frac{1}{2}\int [dk]\, \widetilde{\Gamma}^{aex}_{\mu\alpha\beta}\Delta^{\alpha\rho}_{ee'}(k)\widetilde{\mathbb{\Gamma}}^{be'x'}_{\nu\rho\sigma}\ \Delta^{\beta\sigma}_{xx'}(k+q),$$
$$\widehat{\Pi}^{ab}_{\mu\nu}(q)\big|_{a_2} = \frac{1}{2}\int [dk]\, \widetilde{\Gamma}^{abex}_{\mu\nu\alpha\beta}\Delta^{\alpha\beta}_{ex}(k), \qquad (2)$$

where $[dk] = d^d k/(2\pi)^d$ with $d = 4-\epsilon$ the dimension of space-time. By virtue of the BFM all-order WI

$$q_1^\mu \widetilde{\mathbb{\Gamma}}^{abc}_{\mu\alpha\beta}(q_1,q_2,q_3) = g f^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2) - \Delta^{-1}_{\alpha\beta}(q_3)\right], \qquad (3)$$

and using the tree-level $\widetilde{\Gamma}_{\mu\nu\alpha\beta}$ given in [4], we have

$$q^\nu \widehat{\Pi}^{ab}_{\mu\nu}(q)\big|_{a_1} = C_A\, g^2 \delta^{ab}\, q_\mu \int [dk]\, \Delta^\rho_\rho(k),$$
$$q^\nu \widehat{\Pi}^{ab}_{\mu\nu}(q)\big|_{a_2} = -C_A\, g^2 \delta^{ab}\, q_\mu \int [dk]\, \Delta^\rho_\rho(k), \qquad (4)$$

and thus, $q^\nu(\widehat{\Pi}^{ab}_{\mu\nu}(q)\big|_{a_1} + \widehat{\Pi}^{ab}_{\mu\nu}(q)\big|_{a_2}) = 0$. The importance of this transversality property in the context of the SD equation is that it allows for a meaningful first approximation: instead of the system of coupled equations involving gluon and ghost propagators, one may consider only the subset containing gluons, without compromising the crucial property of transversality. We will therefore study as the first non-trivial approximation for $\widehat{\Pi}_{\mu\nu}(q)$ the diagrams (a_1) and (a_2). Of course, we have no *a priori* guarantee that this particular subset is numerically dominant. Actually, as has been argued in a series of SD studies, in the context of the conventional Landau gauge it is the ghost sector that furnishes the leading contribution [6] Clearly, it is plausible that this characteristic feature may persist within the PT-BFM scheme as well, and we will explore this crucial issue in the near future.

The equation given in (2) is not a genuine SD equation, in the sense that it does not involve the unknown quantity $\widehat{\Delta}$ on both sides. Substituting $\Delta \to \widehat{\Delta}$ on the RHS of (2) (see discussion in [3]), we obtain

$$\widehat{\Pi}_{\mu\nu}(q) = \frac{1}{2} C_A\, g^2 \int [dk]\, \widetilde{\Gamma}^{\alpha\beta}_{\mu}\widehat{\Delta}(k)\widetilde{\mathbb{\Gamma}}_{\nu\alpha\beta}\widehat{\Delta}(k+q)$$
$$- C_A\, g^2\, d\, g_{\mu\nu}\int [dk]\, \widehat{\Delta}(k), \qquad (5)$$

with $\widetilde{\Gamma}_{\mu\alpha\beta} = (2k+q)_\mu g_{\alpha\beta} - 2q_\alpha g_{\mu\beta} + 2q_\beta g_{\mu\alpha}$, and

$$q^\nu \widetilde{\mathbb{\Gamma}}_{\nu\alpha\beta} = \left[\widehat{\Delta}^{-1}(k+q) - \widehat{\Delta}^{-1}(k)\right] g_{\alpha\beta}\,. \qquad (6)$$

We can then linearize the resulting SD equation, by resorting to the Lehmann representation for the scalar part of the gluon propagator [1],

$$\widehat{\Delta}(q^2) = \int d\lambda^2\, \frac{\rho\,(\lambda^2)}{q^2 - \lambda^2 + i\epsilon}\,, \qquad (7)$$

and setting on the first integral of the RHS of eq. (5):

$$\widehat{\Delta}(k)\widetilde{\mathbb{\Gamma}}_{\nu\alpha\beta}\widehat{\Delta}(k+q) = \int \frac{d\lambda^2\, \rho(\lambda^2)\, \widetilde{\Gamma}^{\rm L}_{\nu\alpha\beta}}{[k^2-\lambda^2][(k+q)^2-\lambda^2]}\,, \qquad (8)$$

where $\widetilde{\Gamma}^{\rm L}_{\nu\alpha\beta}$ must be such as to satisfy the tree-level WI

$$q^\nu \widetilde{\Gamma}^{\rm L}_{\nu\alpha\beta} = \left[(k+q)^2 - \lambda^2\right] g_{\alpha\beta} - (k^2-\lambda^2) g_{\alpha\beta}\,. \qquad (9)$$

We propose the following form for the vertex:

$$\widetilde{\Gamma}^{\rm L}_{\nu\alpha\beta} = \widetilde{\Gamma}_{\nu\alpha\beta} + c_1\left((2k+q)_\nu + \frac{q_\nu}{q^2}\left[k^2-(k+q)^2\right]\right) g_{\alpha\beta}$$
$$+ \left(c_3 + \frac{c_2}{2\,q^2}\left[(k+q)^2 + k^2\right]\right)(q_\beta g_{\nu\alpha} - q_\alpha g_{\nu\beta})\,, \qquad (10)$$

which, due to the presence of the massless poles, allows the possibility of infrared finite solution.

Due to the QED-like WIs satisfied by the PT Green's functions, $\widehat{\Delta}^{-1}(q^2)$ absorbs all the RG-logs. Consequently, the product $\widehat{d}(q^2) = g^2\widehat{\Delta}(q^2)$ forms a RG-invariant (μ-independent) quantity. Notice however that eq. (5) does not encode the correct RG behavior: when written in terms of $\widehat{d}(q^2)$ it is not manifestly g^2-independent, as it should. In order to restore the correct RG behavior, we use the simple prescription proposed in [1], whereby we substitute every $\widehat{\Delta}(z)$ appearing on the RHS of the SD equation by

$$\widehat{\Delta}(z) \to \frac{g^2\,\widehat{\Delta}(z)}{\bar{g}^2(z)} \equiv [1 + \tilde{b} g^2 \ln(z/\mu^2)]\widehat{\Delta}(z). \qquad (11)$$

Then, setting $\tilde{b} \equiv \frac{10\, C_A}{48\pi^2}$, $\sigma \equiv \frac{6(c_1+c_2)}{5}$, $\gamma \equiv \frac{4+4\,c_1+3\,c_2}{5}$, we finally obtain

$$\widehat{d}^{\,-1}(q^2) = q^2\left\{K' + \tilde{b}\int_0^{q^2/4} dz\, \left(1-\frac{4z}{q^2}\right)^{1/2} \frac{\widehat{d}(z)}{\bar{g}^2(z)}\right\}$$
$$+\gamma\tilde{b}\int_0^{q^2/4} dz\, z\left(1-\frac{4z}{q^2}\right)^{1/2}\frac{\widehat{d}(z)}{\bar{g}^2(z)} + \widehat{d}^{\,-1}(0), \qquad (12)$$

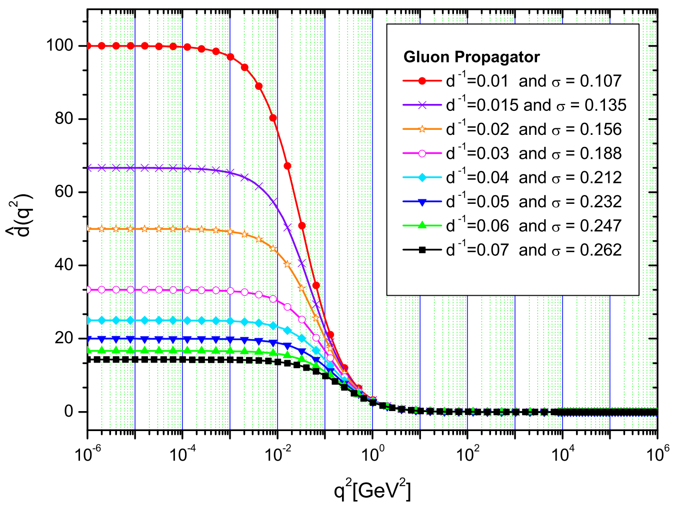

Fig. 2. Results for $\widehat{d}(q^2)$, for different values for $\widehat{d}^{-1}(0)$ (all in GeV^2), and the corresponding values for σ.

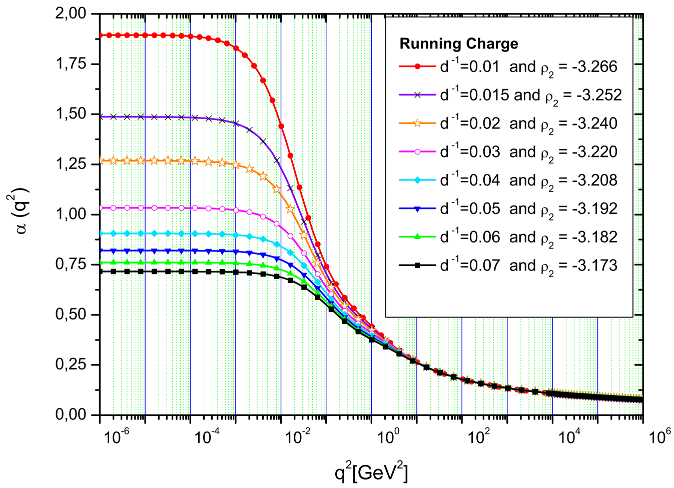

Fig. 3. $\alpha(q^2) = \overline{g}^2_{\rm NP}(q^2)/4\pi$ from the propagators of fig. 2.

$$K' = \frac{1}{g^2} - \tilde{b}\int_0^{\mu^2/4} {\rm d}z \left(1+\gamma\frac{z}{\mu^2}\right)\left(1-\frac{4z}{\mu^2}\right)^{1/2}\frac{\widehat{d}(z)}{\overline{g}^2(z)}\,, \tag{13}$$

and

$$\widehat{d}^{-1}(0) = -\frac{\tilde{b}\sigma}{\pi^2}\int {\rm d}^4k\,\frac{\widehat{d}(k^2)}{\overline{g}^2(k^2)}\,. \tag{14}$$

It is easy to see now that eq. (12) yields the correct UV behavior, *i.e.* $\widehat{d}^{-1}(q^2) = \tilde{b}\,q^2\ln(q^2/\Lambda^2)$.

When solving (12) we will be interested in solutions that are qualitatively of the general form [1]

$$\widehat{d}(q^2) = \frac{\overline{g}^2_{\rm NP}(q^2)}{q^2+m^2(q^2)}\,, \tag{15}$$

where

$$\overline{g}^2_{\rm NP}(q^2) = \left[\tilde{b}\ln\left(\frac{q^2+f(q^2,m^2(q^2))}{\Lambda^2}\right)\right]^{-1}, \tag{16}$$

$\overline{g}^2_{\rm NP}(q^2)$ represents a non-perturbative version of the RG-invariant effective charge of QCD: in the deep UV it goes over to $\overline{g}^2(q^2)$, while in the deep IR it "freezes" [1,7], due to the presence of the function $f(q^2,m^2(q^2))$, whose form will be determined by fitting the numerical solution. The function $m^2(q^2)$ may be interpreted as a momentum-dependent "mass". In order to determine the asymptotic behavior that eq. (12) predicts for $m^2(q^2)$ at large q^2, we replace eq. (15) on both sides, set $(1-4z/q^2)^{1/2}\to 1$, obtaining self-consistency provided that

$$m^2(q^2) \sim m_0^2\ln^{-a}\left(q^2/\Lambda^2\right), \quad \text{with} \quad a = 1+\gamma > 0. \tag{17}$$

The seagull-like contributions, defining $\widehat{d}^{-1}(0)$ in (14), are essential for obtaining IR finite solutions. However, the integral in (14) should be properly regularized, in order to ensure the finiteness of such a mass term. Recalling that in dimensional regularization $\int[{\rm d}k]/k^2 = 0$, we rewrite eq. (14) (using (15)) as

$$\begin{aligned}\widehat{d}^{-1}(0) &\equiv -\frac{\tilde{b}\sigma}{\pi^2}\int[{\rm d}k]\left(\frac{\overline{g}^2_{\rm NP}(k^2)}{[k^2+m^2(k^2)]\overline{g}^2(k^2)}-\frac{1}{k^2}\right)\\ &= \frac{\tilde{b}\sigma}{\pi^2}\int[{\rm d}k]\frac{m^2(k^2)}{k^2[k^2+m^2(k^2)]}\\ &+\frac{\tilde{b}^2\sigma}{\pi^2}\int[{\rm d}k]\,\widehat{d}(k^2)\ln\left(1+\frac{f(k^2,m^2(k^2))}{k^2}\right). \end{aligned}\tag{18}$$

The first integral converges provided that $m^2(k^2)$ falls asymptotically as $\ln^{-a}(k^2)$, with $a>1$, while the second requires that $f(k^2,m^2(k^2))$ should drop asymptotically at least as fast as $\ln^{-c}(k^2)$, with $c>0$. Notice that perturbatively $\widehat{d}^{-1}(0)$ vanishes, because $m^2(k^2)=0$ to all orders, and, in that case, $f=0$ also.

Solving numerically eq. (12), subject to the constraint of eq. (14), we obtain solutions shown in fig. 2; they can be fitted perfectly by means of a running coupling that freezes in the IR, shown in fig. 3, and a running mass that vanishes in the UV [3]. σ is treated as a free parameter, whose values are fixed in such a way as to achieve compliance between eqs. (12)–(14).

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